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FURTHER STUDY ON THE KINETIC ENERGY BALANCE [1, 2]

ERNEST C. KUNG

Department of Atmospheric Science, University of Missouri, Columbia, Mo.

ABSTRACT

Various aspects of our studies of the kinetic energy balance are discussed, using an improved scheme of computing the horizontal and vertical transports of the kinetic energy with the observed wind and geopotential data over North America. On a firmer basis than before, it has been shown that there is a considerable amount of kinetic energy dissipated outside the planetary boundary layer, particularly at the jet-stream level. This also may imply that intensity of the atmospheric general circulation is significantly higher than is being assumed in most of the numerical models of the atmosphere.

1. INTRODUCTION

In a series of previous reports (Kung, 1966*a*, 1966*b*, 1967), studies of the large-scale balance of the kinetic energy in the atmosphere were reported. One of the main objectives in this series of studies is to establish the basic observational facts about the kinetic energy generation and dissipation with the standard synoptic observational data. Concerning the estimate of energy dissipation, we particularly pointed out that *we must compute the dissipation without employing specific theories about the dissipation mechanism*. Our argument is that the independently computed dissipation values, free of specific theories or conventional assumptions, can provide an independent observational basis to study the energy dissipation, since at present virtually no consensus exists in regard to the problem of energy dissipation.

One obvious and probably the most direct way to do this is to obtain the dissipation as the residual term to balance other energy parameters in the kinetic energy equation that are computed with the observed wind and geopotential data. In this way we can also study the energy balance without using the controversial vertical p-velocity ω in the generation term. An adequate numerical analysis scheme and a dense, yet large enough, network of radiosonde observation are two essential requirements for this task. In the series of previous reports (loc. cit.), a proper scheme of computation was developed and applied to the daily 00 GMT and 12 GMT rawinsonde/radiosonde observations up to a 5-yr period from May 1958 to April 1963 over the North American Continent, obtained from the Massachusetts Institute of Technology (MIT) General Circulation Data Library (NSF Grants GP 820 and GP 3657). Those reports indicated that the kinetic energy generation from the work done by the horizontal pressure force and the dissipation are both maximum in the planetary boundary layer and at the jet-stream level. While the majority of the earlier and current numerical weather prediction models assume that the dissipation takes place only in the planetary boundary layer, this could be one of the major items yet to be incorporated in those models. Our computed dissipation value also seems much higher than that generally obtained from studies with the conventionally computed ω. If the magnitude of our dissipation value is correct, it should indicate a more intense energy cycle or efficient atmospheric engine than is generally assumed at present. Intriguing diurnal and seasonal variations (see also Kung and Soong, 1969) were also observed.

During our observational study of the energy balance, discussions and references concerning our reports (loc. cit.) were made in personal communications and in publications. Smagorinsky, Manabe, and Holloway (1965) showed in the numerical experiment of their Geophysical Fluid Dynamics Laboratory (GFDL) model that there were two maxima of generation and dissipation in the boundary layer and at the jet-stream level. Importance of the dissipation in the free atmosphere was also indicated by the fact (see Trout and Panofsky, 1969) that numerical predictions by the frictionless equations are subject to large errors near the tropopause. Dutton and Johnson (1967) found the diabatic generation rate of the zonal available potential energy from their exact theory to be very close to the dissipation given by us and argue that the atmospheric energy transformations are more intense than generally realized. Wiin-Nielsen (1967, 1968) emphasized the difference between our large dissipation value and conventionally accepted smaller values, the deviation of our finding from the widely employed assumption that most of the dissipation is accounted for in the boundary layer, and also the consequent implications for general circulation energetics. Ellsaesser (1968) showed that inclusion of the dissipation mechanism in the free atmosphere is necessary for numerical weather prediction models. Ellsaesser (1969) further examined our diagnosis of the kinetic energy balance from various aspects in his climatological study of the energy dissipation and agreed with our major conclusions. Recently, Trout and Panofsky (1969) estimated the energy dissipation near the tropopause from clear-air turbulence spectra measurements and clear-air turbulence probabilities and found very close

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agreement with our dissipation value (Kung, 1966*b*) in that portion of the atmosphere.

However, some important points were left out in our previous reports (Kung, 1966*a*, 1966*b*, 1967). A likely overestimate of the horizontal outflow of the kinetic energy and the employment of the continental-scale divergence in computing the vertical transport of the kinetic energy should be examined. The effect of the radiation error on the radiosonde data ought to be scrutinized. A seemingly large summer dissipation value in comparison with the winter value, interpretation of the computed dissipation value, and consistency of our reports are also to be examined in view of the improvement in the computation of transport terms. As our study of the atmospheric energy balance is entering a new phase (see section 5) with support of the National Science Foundation (NSF Grant GA-1287), it is intended in this paper to study the above-mentioned points to conclude the preliminary phase of our study.

2. SCHEME OF COMPUTATIONAL ANALYSIS

As described in the previous reports (loc. cit.), the kinetic energy dissipation $\overline{E}$ over the continental area is obtained by

$$-\overline{E}=\frac{\partial\overline{k}}{\partial t}+\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds+\frac{\partial\overline{\omega k}}{\partial p}+\overline{\mathbf{V}\cdot\nabla\phi} \qquad (1)$$

where $\partial\overline{k}/\partial t$ is the local change of the kinetic energy, $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ the horizontal outflow, $\partial\overline{\omega k}/\partial p$ the vertical transport, and $-\overline{\mathbf{V}\cdot\nabla\phi}$ the generation. The horizontal bar notation indicates the continental area mean. A and s are the continental area and boundary, $\mathbf{n}$ is the unit vector normal to s, and other symbols are those of the standard hemispherical polar coordinate (x, y, p, t) system. The technique of evaluating the cross-isobar flow with the observed wind $\mathbf{V}$ and geopotential ϕ was described in a previous paper (Kung, 1966*a*). The vertical p-velocity ω is obtained by kinematically integrating the continuity equation

$$\omega_{p_1}=\int_{p_1}^{p_2}\nabla\cdot\mathbf{V}dp+\omega_{p_2} \qquad (2)$$

assuming $\omega=0$ at the surface.

In the previous studies (loc. cit.) the kinetic energy, obtained with the observed wind data at a single station on the continental boundary, was used along with those of other stations on the boundary to compute the horizontal outflow. When the wind is strong, the error component in the wind observation, which is squared in the process of obtaining k, had no way to cancel out with that from other stations and tended to give a systematic overestimate of $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$. In a preceding report (Kung, 1968), it was demonstrated that, simply by taking the averaged value of k of several nearby stations instead of that of a single station, $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ value for winter was significantly reduced at the jet-stream level. After a series of tests, a simple scheme of normalizing k in computing the line integral $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ was adopted. The method is to obtain the weighted mean k by weighing the stronger wind by ⅓ and weaker wind by ⅔ for two neighboring stations on the continental boundary. The reason for giving more weight to the weaker wind of the two neighboring stations is to weigh the variation of the wind field more smoothly than the simple average wind speed to give the normalized k. Actually, our careful case studies verified that the line integral value of $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ over the North American Continent was nearly the same, either using the weighted mean wind speed for k as described above or using the simple average of two wind speeds. It is further verified that other simple methods for normalizing k all gave about the same reduced value of $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$. Apparently the previous overestimate of the horizontal outflow due to the random error in the wind observation can be corrected by such normalizing processes, although some further deduction of this term might still be possible by a sophisticated normalizing process such as applying the binomial smoother to the normal wind component to the boundary.

As an example, figure 1 compares the computed horizontal outflow with the present normalized k and the previous unnormalized k for 00 GMT of February 1962 and 12 GMT of January 1963. It is clear that previously we overestimated the horizontal outflow and thus underestimated the dissipation as the residual term at the jet-stream level during the winter (also see table 1). However, during the summer 6 mo, no overestimate was

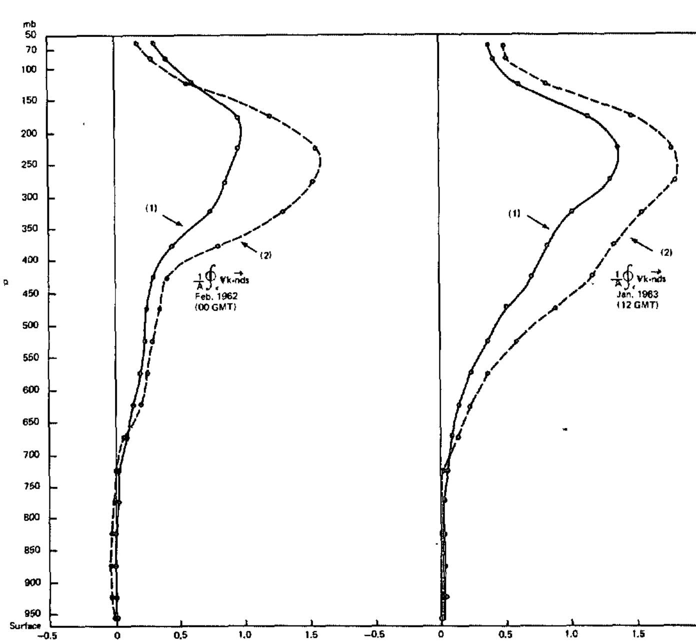

FIGURE 1.—Comparison of the horizontal outflow of kinetic energy computed with the present scheme (1) and previous scheme (2).

TABLE 1.—*Comparison of estimated values of the horizontal and vertical transports of kinetic energy with the previous and present schemes of computation (00 and 12* GMT *average). Parameters are integrated from surface to 50 mb and in units of watts* m^{-2}.

Season	$\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$		$\partial\overline{\omega k}/\partial p$	
	Previous	Present	Previous	Present
Winter 6 mo. (Nov.–Apr.)	5.818	5.190	−1.106	−0.062
Summer 6 mo. (May–Oct.)	0.222	0.367	−.028	.019
Annual mean	3.020	2.778	−.567	−.021

TABLE 2.—*Total, mean, and eddy vertical transport of kinetic energy in different portions of the atmosphere over the North American Continent during the 1-yr period in units of watts* m^{-2} *(00 and 12* GMT *average)*

Pressure layer (mb)	$\partial\overline{\omega k}/\partial p$	$\partial\bar{\omega}\bar{k}/\partial p$	$\partial\overline{\omega' k'}/\partial p$
Boundary layer	−0.003	−0.002	−0.001
*968–700	−.021	−.010	−.011
700–400	.057	−.137	.194
400–50	−.057	−.420	.363
Total (surface–50)	−.021	−.567	.546

*Continental mean surface pressure

made. The overestimate of outflow due to this type of error may be significant only when the wind speed is very large during the winter.

Concerning the vertical transport, $\bar{\omega}\bar{k}$ was substituted for $\overline{\omega k}$ in computing $\partial\overline{\omega k}/\partial p$ in the previous reports (loc. cit.). The underlying assumption here was that $\partial\overline{\omega k}/\partial p$ seemed to be a small term in the kinetic energy balance and should not upset the tentative conclusions. Nevertheless, from physical reasoning the representativeness of $\bar{\omega}\bar{k}$ in the substitution for $\overline{\omega k}$ was doubtful. Moreover, in discussing the vertical profile of the kinetic energy balance, the vertical transport term $\partial\omega k/\partial p$ is the only term available for redistribution of the generated kinetic energy at a particular level for the dissipation at some other levels of the atmosphere.

In this study the vertical velocity ω was computed by equation (2) for each individual station over the continent with the locally evaluated divergence of the grid scale. The local divergence was evaluated by a scheme which is a modification of Bellamy's triangle method (1949). In our computation schemes, the four to six most closely located surrounding stations are considered. Four to six triangles were then formed; each one with two neighboring stations, B and C, and that particular station A in the center. Then with the assumption of linear variation of the wind field, two sets of simultaneous equations

$$\Delta u_{BA}=\frac{\partial u}{\partial x}\Delta X_{BA}+\frac{\partial u}{\partial y}\Delta y_{BA}$$

$$\Delta u_{CA}=\frac{\partial u}{\partial x}\Delta X_{CA}+\frac{\partial u}{\partial y}\Delta y_{CA} \tag{3}$$

and

$$\Delta(v\cos\theta)_{BA}=\frac{\partial v\cos\theta}{\partial x}\Delta X_{BA}+\frac{\partial v\cos\theta}{\partial y}\Delta y_{BA}$$

$$\Delta(v\cos\theta)_{CA}=\frac{\partial v\cos\theta}{\partial x}\Delta X_{CA}+\frac{\partial v\cos\theta}{\partial y}\Delta y_{CA} \tag{4}$$

were solved for $\partial u/\partial x$ and $\partial v\cos\theta/\partial y$ to obtain the divergence

$$\nabla\cdot\mathbf{V}=\frac{\partial u}{\partial x}+\frac{1}{\cos\theta}\frac{\partial v\cos\theta}{\partial y}$$

where θ is the latitude. The obtained $\nabla\cdot\mathbf{V}$ values for each triangle were then added up with the area of the triangles as the weighting factor. The value is then the local divergence for computation of ω.

It is well known that the kinematically estimated ω with the divergence computed by Bellamy's triangle method works well in the lower and midtroposphere. However, due to the accumulation of error of the computed divergence during the vertical integration of the continuity equation, ω may spuriously deviate from zero at the top of the atmosphere if integrated from the lower boundary. One significant source of error in the divergence estimate, according to Schmidt and Johnson (1967), is that nonlinear variation of the wind field becomes significant in the upper troposphere when certain circulation patterns of the jet stream exist. In our modified scheme, as described above, the error in divergence is reduced to a certain extent because several divergence values of adjacent triangles around a station were added up to obtain a value of local divergence. Also, since we are only interested in $\partial\overline{\omega k}/\partial p$, a certain part of the accumulating error in the vertical integration can be expected to drop out in taking the vertical divergence of the $\overline{\omega k}$. Nevertheless, to the ω values at the levels above 400 mb, Kurihara's correction scheme (1961) for ω was employed in this study by requiring ω at the top of the atmosphere to be zero and adopting his correction coefficient $\epsilon'(p)$ above 400 mb. It should be emphasized here that the correction was applied to ω only above 400 mb. Thus in discussing the vertical transport between upper and midtroposphere, we need not worry about the effect of correction on ω. Our case study also indicates that the effect of the correction above 400 mb on $\partial\overline{\omega k}/\partial p$ is not significant for our scale of analysis.

Table 2 compares the vertical transport computed in this study $\partial\overline{\omega k}/\partial p$ and that of the previous study $\partial\bar{\omega}\bar{k}/\partial p$ along with their difference $\partial\overline{\omega' k'}/\partial p$. The computed values of $\partial\overline{\omega k}/\partial p$ and $\partial\bar{\omega}\bar{k}/\partial p$ are quite different especially above the lower troposphere. It is significant that the vertical transport thus recomputed in this study becomes an order of magnitude smaller than we obtained previously. As shown in table 1 and 2, when $\partial\overline{\omega k}/\partial p$ is integrated from the surface to 50 mb, the previously obtained large downward inflow of the kinetic energy during winter from the layer above 50 mb, which was rather unlikely, essentially disappeared.

Before proceeding to discuss the kinetic energy balance, the scheme of computing the generation $-\overline{\mathbf{V}\cdot\nabla\phi}$ used throughout our series of reports (loc. cit.) and in this paper should be examined with regard to the diurnal variation. As reported previously (Kung, 1967), the generation and dissipation seem to be consistently greater at 00 GMT than at 12 GMT. It is not difficult to expect an organized large-scale variation of the vertical motion and divergence patterns and, consequently, the significant diurnal variation of the kinetic energy budget from the diurnal cycle of surface heating in the lower and midtroposphere (see Curtis and Panofsky, 1958, and Bleeker and Andre, 1951). The diurnal variation above the midtroposphere, on the other hand, is intriguing as well as puzzling. Examination of the intermediate output of our computation indicates that the diurnal variation of the wind affects the value of $-\overline{\mathbf{V}\cdot\nabla\phi}$ more sensitively than that of $\nabla\phi$. Accumulating studies in the diurnal wind variation are not inconsistent with the diurnal variation of $-\overline{\mathbf{V}\cdot\nabla\phi}$. The works by Darkow and Thompson (1968), Finger, Harris, and Teweles (1965), Harris, Finger, and Teweles (1966), Hering and Borden (1962), and Wallace and Hartranft (1969) may be cited in this connection.

It is well recognized that the radiation error becomes appreciable in the reported stratospheric temperature and height data. The National Meteorological Center, ESSA, has been systematically investigating the problem as summarized by McInturff and Finger (1969). During the period 1959 through 1963, which corresponds with the period of our data source, the correction for the radiation error was applied in most cases, at the stations, to radiosonde observations of duct-type instruments, which were mostly used. The necessary further correction to those reported data was listed by Finger, Woolf, and Anderson (1965) for solar elevation from 0° to 90° from the 100-mb level to the 10-mb level. It is reasonable to assume some radiation error below 100 mb. However, in view of the listing by Finger et al. (loc. cit.), the needed correction seems rather small below that level. In fact, the information for correction of radiation error below the 100-mb level is not available (personal communication with R. M. McInturff). Although most of the atmospheric layer we are interested in is below the 100-mb level and our results obtained would not be influenced too much by the radiation error above that level, it is interesting to examine the sensitivity of our $-\overline{\mathbf{V}\cdot\nabla\phi}$ computation to this type of error in the data.

The error contained in the height report accumulates at the upper levels through addition of the biased thickness from below. However, most of the systematic error of this type will disappear by taking the gradient $\nabla\phi$ using the nearby stations whose observational errors due to radiation are supposedly very similar. The radiation error in the thickness also affects the reported value of the wind speed and direction. In this regard, as stated by Finger, Harris, and Teweles (1965), it is important to turn our attention to the general recognition that the wind observations are not known to have systematic errors. This should mean that the wind error due to the thickness report is too small to be recognized as the systematic cause of error in comparison with other error sources of random nature. To gain some insight into the radiation error range, several cases of original wind-aloft computation data at Columbia, Mo., were obtained for mid-July of 1968; the wind speed and direction at 100-, 70-, and 50-mb levels were recomputed, assuming an extreme value of radiation error in the thickness report. The assumed radiation errors result in thickness of 10 m per 2-min reading at 100 mb, 18 m at 70 mb, and 27 m at 50 mb. The recomputation shows that the difference between corrected and originally reported wind speed is less than 5 percent of the reported wind at 100 mb and 5 to 10 percent at 70 mb and 50 mb, unless the wind is extremely light (i.e., less than 3 m sec^{-1}), in which case no spurious diurnal variation in this term is detectable anyway. The corresponding difference in the wind direction is mostly less than 3° (often within 1°), again unless the wind is extremely light.

To test the sensitivity of our $-\overline{\mathbf{V}\cdot\nabla\phi}$ computation to the extreme radiation error in the wind data, four cases were illustrated with the 00 GMT observations in July 1962, as shown in figure 2. Case (1) is the original computation without any correction on wind data. In case (2), the u-component of the wind is increased by 10 percent at 500-, 450-, 400-, 350-, and 300-mb levels; by 15 percent at 250- and 200-mb levels; by 20 percent at 150- and 100-mb levels; and by 10 percent at 70- and 50-mb levels. In case (3), the u-component was reduced by the same percentage as case (2) at the corresponding levels; in case (4), the v-component was increased by the same percentage; and in case (5), the v-component was reduced by the same percentage. Figure 2 thus clearly shows that the computed vertical profile of $-\overline{\mathbf{V}\cdot\nabla\phi}$ will respond in some way to the radiation error components in the wind report. However, even with the extremely large radiation error component, which is very unlikely below the 100-mb level, the effects on the $-\overline{\mathbf{V}\cdot\nabla\phi}$ profile do not seem to be large enough to change our observations on the diurnal variation.

3. OBSERVED KINETIC ENERGY BALANCE

In this study the horizontal and vertical transport terms, $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ and $\partial\overline{\omega k}/\partial p$, were recomputed in accordance with the improved scheme, as discussed in the preceding section. The data source was the same twice-a-day wind and geopotential observations over the North American Continent from the MIT General Circulation

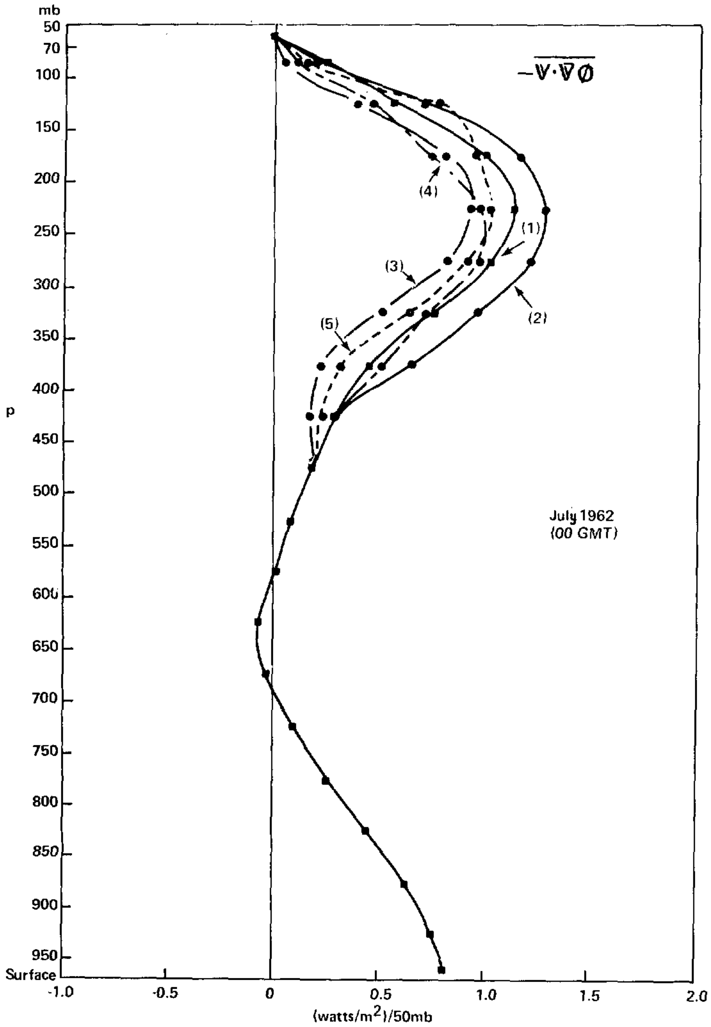

FIGURE 2.—Sensitivity of computation of the generation term to the extreme radiation error in the wind observation. See the text for the explanation of (1) through (5).

Data Library (see section 1) that we used before. However, the period of the recomputation only covered from January 1962 through January 1963. December 1962 data were not utilized because of the technical input difficulty of the data tapes in our possession for that part. For other details of the data handling, reference may be made to one of the previous reports (Kung, 1967). Throughout this paper, unless otherwise specified, the winter 3-mo value of a computed parameter is the average of the daily value of January, February 1962 and January 1963; the summer 3-mo value of June, July, and August 1962; the winter 6 mo of January, February, March, April, and November 1962 and January 1963; the summer 6-mo value of May, June, July, August, September, and October 1962; and the annual mean of the average over those 12 mo.

The kinetic energy balance with the recomputed horizontal and vertical transport terms is summarized in table 3 for the average of 00 and 12 GMT observations.

TABLE 3.—*Kinetic energy balance in different portions of the atmosphere in units of watts m^{-2} (00 and 12 GMT average)*

Season	Pressure layer (mb)	$\partial\bar{K}/\partial t$	$\frac{1}{A}\oint_c Vk\cdot nds$	$\partial\overline{\omega k}/\partial p$	$\overline{\mathbf{V}\cdot\nabla\phi}$	$\bar{E}$
	Boundary layer*	0.003	0.033	−0.015	3.107	3.086
Winter 3 mo (Dec.–Feb.)	**969–700	.008	.154	−.064	5.134	5.034
	700–400	.124	1.619	.580	3.069	.746
	400–50	.301	6.689	−.618	9.293	2.922
	Total	.433	8.462	−.102	17.496	8.702
	Boundary layer*	.002	−.006	.021	1.121	1.104
Summer 3 mo (June–Aug.)	**969–700	.003	.012	.042	2.045	1.987
	400–400	.000	.114	−.088	1.040	1.014
	700–50	.009	.479	.089	2.068	1.491
	Total	.012	.605	.043	5.153	4.493
	Boundary layer*	−.001	.023	.000	2.696	2.674
Winter 6 mo (Nov.–Apr.)	**969–700	−.012	.081	−.047	4.492	4.469
	700–400	.014	.875	.169	1.933	.875
	400–50	.043	4.233	−.184	5.432	1.339
	Total	.045	5.190	−.062	11.857	6.683
	Boundary layer*	−.000	−.003	−.005	1.372	1.381
Summer 6 mo (May–Oct.)	**969–700	−.002	.010	.005	2.417	2.404
	700–400	−.011	.043	−.056	.922	.945
	400–50	−.019	.313	.069	1.501	1.137
	Total	−.032	.367	.019	4.839	4.486
	Boundary layer*	−.001	.010	−.003	2.034	2.027
Annual mean	**969–700	−.007	.046	−.021	3.455	3.436
	700–400	.001	.459	.057	1.427	.910
	400–50	.012	2.273	−.057	3.466	1.238
	Total	.007	2.778	−.021	8.348	5.584

* 969–868 mb
** Continental mean surface pressure

The lowest 100-mb layer of the atmosphere is regarded as the planetary boundary layer. The atmospheric layer below 700 mb, including the boundary layer, may be regarded as the lower troposphere; the layer between 700 and 400 mb as the midtroposphere; and the layer between 400 and 50 mb as the jet-stream level, although during summer the actual position of the jet stream is at the lower portion of this layer. With the recomputed $\partial\overline{\omega k}/\partial p$ instead of the previous $\partial\overline{\omega k}/\partial p$, the picture presented in this table confirms our previous suggestion that the kinetic energy generation and the dissipation have double maxima in the boundary layer and at the jet stream. The vertical transport into or out of the boundary layer or any of the three atmospheric layers as listed in the table is negligibly small in comparison with the generation value in each layer. Especially noteworthy is that $\partial\overline{\omega k}/\partial p$ shows the smallest value in the boundary layer and that, although small, during the winter and also on an annual basis the midtroposphere exports the kinetic energy to the jet-stream level. This inevitably leads to a strong argument that the kinetic energy generated in the free atmosphere cannot be carried down to the boundary layer for

TABLE 4.—*Kinetic energy generation* $-\overline{\mathbf{V}\cdot\nabla\phi}$ *within each pressure layer in units of watts* m^{-2}

Pressure layer (mb)	Winter 6 mo (Nov.-Apr.)		Summer 6 mo (May-Oct.)		Annual mean	
	00 GMT	12 GMT	00 GMT	12 GMT	00 GMT	12 GMT
969*-950	0.637	0.599	0.323	0.262	0.480	0.431
950-900	1.635	1.266	.931	.556	1.283	.911
900-850	1.201	.821	.791	.294	.996	.558
850-800	.792	.505	.563	.210	.677	.357
800-750	.522	.341	.358	.175	.440	.258
750-700	.370	.294	.231	.140	.300	.217
700-650	.274	.257	.170	.136	.222	.197
650-600	.174	.263	.158	.121	.166	.192
600-550	.106	.333	.138	.137	.122	.235
550-500	.142	.443	.135	.168	.139	.306
500-450	.223	.575	.164	.169	.193	.372
450-400	.326	.750	.207	.140	.267	.445
400-350	.533	.912	.399	.085	.466	.499
350-300	.739	.872	.709	.083	.724	.477
300-250	.809	.761	.886	.025	.848	.393
250-200	.862	.655	.770	−.226	.816	.215
200-150	1.031	.504	.571	−.436	.801	.034
150-100	1.113	.421	.399	−.375	.756	.023
100-70	.701	.270	.144	−.099	.423	.086
70-50	.496	.183	.051	.015	.273	.099

*Continental mean surface pressure

TABLE 5.—*Kinetic energy dissipation* $\bar{E}$ *within each pressure layer in units of watts* m^{-2}

Pressure layer (mb)	Winter 6 mo (Nov.-Apr.)		Summer 6 mo (May-Oct.)		Annual mean	
	00 GMT	12 GMT	00 GMT	12 GMT	00 GMT	12 GMT
969*-950	0.637	0.630	0.331	0.282	0.484	0.456
950-900	1.574	1.274	.929	.559	1.251	.916
900-850	1.202	.785	.784	.281	.993	.533
850-800	.809	.492	.559	.192	.684	.342
800-750	.528	.339	.359	.142	.443	.240
750-700	.378	.289	.244	.145	.311	.217
700-650	.180	.248	.202	.132	.191	.190
650-600	.078	.166	.184	.064	.131	.115
600-550	.008	.212	.197	.158	.102	.185
550-500	.101	.204	.115	.175	.108	.190
500-450	.141	.029	.159	.163	.150	.096
450-400	.200	.183	.253	.087	.227	.135
400-350	.171	.568	.512	.034	.341	.301
350-300	.461	.296	.710	−.010	.585	.143
300-250	.085	−.048	.851	−.106	.468	−.077
250-200	−.040	.097	.650	−.287	.305	−.095
200-150	.056	−.241	.340	−.337	.198	−.289
150-100	.365	−.028	.234	−.433	.300	−.231
100-70	.486	.000	.122	−.066	.303	−.033
70-50	.392	.058	.040	.022	.216	.040

*Continental mean surface pressure

dissipation—it must be dissipated in the free atmosphere. Also, there is a significant energy dissipation at the jet-stream level as well as in the boundary layer. As the vertical motion was computed with the grid-scale divergence, no information is available for the transport by the subgrid-scale vertical motion. However, the numerical experiment of the general circulation by Smagorinsky, Manabe, and Holloway (1965), which parameterized the small-scale vertical mixing in the boundary layer with the mixing length concept, also shows the consistent result of $\partial\overline{\omega k}/\partial p$ term with our observation.

The recomputed horizontal outflow $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ resulted in somewhat reduced export and hence the higher dissipation value during the winter. The ratio of the total dissipation between the three winter and summer months is 1.00:0.52, and 1.00:0.67 between six winter and summer months as implied by table 3, which seems to be reasonable in view of the seasonal difference in the intensity of the circulation. Also, as indicated by table 3, the boundary-layer dissipation is 36 percent of the total dissipation on the annual basis. During the winter, the proportion of the dissipation in the boundary layer seems to be larger than that during the summer.

Tables 4 and 5, respectively, list the generation and dissipation within 20 pressure layers from the surface to 50 mb for 6 winter months and 6 summer months and also on an annual basis separately for 00 and 12 GMT observations. Table 6 lists the annual mean kinetic energy balance among energy parameters of equation (1) for 00 and 12 GMT averages. Figure 3 contrasts the winter and summer vertical profiles of the generation and dissipation for 00 and 12 GMT averages. Figure 4 further illustrates the balance of generation and dissipation in the five atmospheric layers during the winter and summer halves of the year and also on an annual basis for 00 and 12 GMT averages. Our improved computation scheme for the transport terms resulted in essential confirmation of our previous arguments (Kung, 1966*a*, 1966*b*, 1967), and explanations on those illustrations will not be needed.

TABLE 6.—*Annual mean kinetic energy budget within each pressure layer in units of watts* m^{-2} *(00 and 12* GMT *average)*

Pressure layer (mb)	$\partial\bar{k}/\partial t$	$\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$	$\partial\overline{\omega k}/\partial p$	$-\overline{\mathbf{V}\cdot\nabla\phi}$	$\bar{E}$
969-950	−0.000	0.002	−0.016	0.455	0.470
950-900	−.000	.006	.007	1.097	1.084
900-850	−.000	.005	.009	.777	.763
850-800	−.002	.006	.000	.517	.513
800-750	−.002	.011	−.002	.349	.342
750-700	−.002	.017	−.021	.259	.264
700-650	−.002	.029	−.009	.209	.191
650-600	.001	.047	.009	.179	.123
600-550	.001	.067	−.033	.179	.144
550-500	.001	.084	−.012	.222	.149
500-450	.000	.104	.055	.283	.123
450-400	.001	.128	.046	.356	.181
400-350	.002	.164	−.005	.482	.321
350-300	.005	.245	−.013	.601	.364
300-250	.003	.394	.029	.620	.195
250-200	−.000	.527	−.117	.515	.105
200-150	.001	.476	−.014	.417	−.045
150-100	.002	.292	.061	.390	.035
100-70	.001	.110	.008	.254	.135
70-50	−.001	.065	−.006	.186	.128

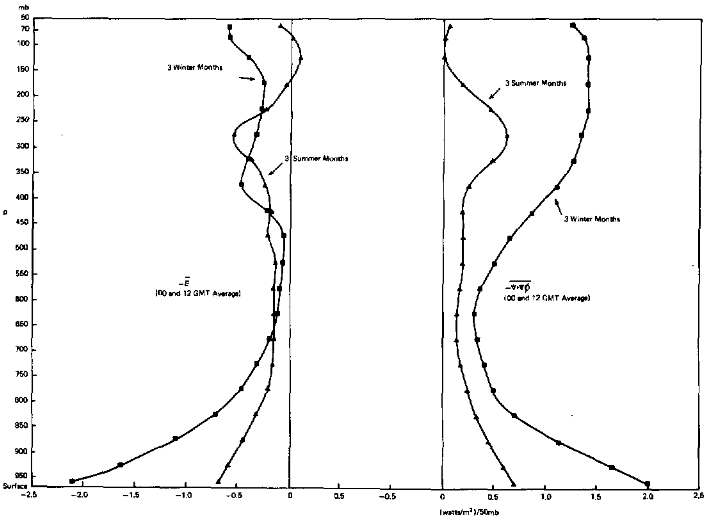

FIGURE 3.—Comparison of the winter and summer vertical profiles of the kinetic energy generation $-\overline{\mathbf{V}\cdot\nabla\phi}$ and dissipation $\overline{E}$.

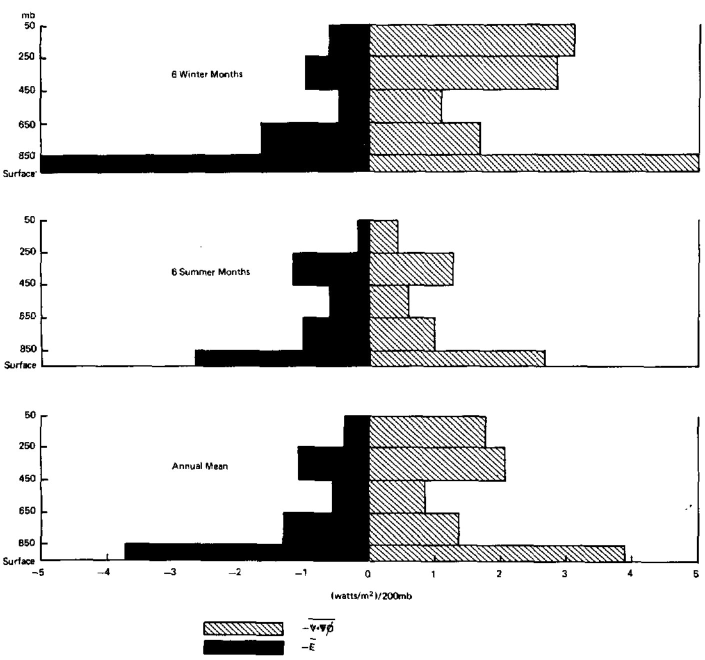

FIGURE 4.—Kinetic energy generation $-\overline{\mathbf{V}\cdot\nabla\phi}$ and dissipation $\overline{E}$ in different portions of the atmosphere.

As illustrated by Krueger, Winston, and Haines (1965) and Kung (1967), the year-to-year variation of the atmospheric energy is significant. However, for our present purpose of confirming our arguments with the recomputed transport terms, the use of 1-yr data should be satisfactory. With a reasonable assumption that the same proportion of reduction in the magnitude of $\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$ and $\partial\overline{\omega k}/\partial p$ during the winter half of the year as recomputed in this study can be applied to the previous computation for the 5-yr period (see table 4 of Kung, 1967), the multiannual mean kinetic energy balance from the surface to 50 mb should read as in table 7.

TABLE 7.—*Estimate of vertically integrated multiannual mean kinetic energy balance from the surface to 50 mb in units of watts* m^{-2}

Season	$\frac{1}{A}\oint_c \mathbf{V}k\cdot\mathbf{n}ds$	$\partial\overline{\omega k}/\partial p$	$-\overline{\mathbf{V}\cdot\nabla\phi}$	$\overline{E}$
Winter	4.48	−0.02	9.64	5.18
Summer	1.71	−.01	4.98	3.28
Annual mean	3.10	−.01	7.31	4.32

4. DISCUSSION

Our study of the kinetic energy balance is based on the synoptic observational network. Thus, the energy dissipation obtained is the dissipation we observe with the grid-scale synoptic data. For this reason the dissipation we obtained may be interpreted as the net energy cascade from the observable scale to the higher wave number. If we assume that there is no net accumulation of the kinetic energy in subgrid scales and the generation term does not play a significant role in those scales, then this net energy cascade should be identical to implied eventual viscous dissipation. As evidenced by some of the negative dissipation values (see table 5), the presence of some energy-generating subgrid scale eddy is probable. If this is the case, we *underestimated* the dissipation; the true dissipation should be even larger than we estimated. However, it is unlikely that we have overestimated the dissipation.

Our study, until now, has been limited to the data coverage over the North American Continent. It is doubtful that the dissipation over the ocean is much smaller than what we have estimated over the continent since the roughness over the ocean is much smaller than that over the land. A careful examination of the problem seems to negate this possibility at least at this moment. As indicated in the author's earlier study (Kung, 1963) concerning the boundary-layer dissipation and also as implied by his recent study of the surface stress (Kung, 1968*a*), the boundary-layer dissipation over the ocean is comparable with that over the continent. This is due to the fact that, while the roughness is small over the ocean, the strong surface wind over the ocean contributes effectively to the boundary-layer dissipation. Above the boundary layer, there is no plausible reason to argue for a smaller dissipation over the ocean. As a matter of fact, our energy-balance study over the North American Continent shows that a considerable amount of kinetic energy generated over the continent is exported at the jet-stream level to the North Atlantic Ocean, suggesting that this amount of kinetic energy is likely to be dissipated in the free atmosphere over the ocean in addition to the kinetic energy generated there.

At this point it will be interesting to review some recent works on the problem of energy dissipation in the

TABLE 8.—*Comparison of the dissipation rate $\bar{E}$ in units of watts m^{-2} near the tropopause with studies by Trout and Panofsky (1969) and Ellsaesser (1968)*

Layer (ft)	Trout and Panofsky	Ellsaesser	This study	
			00 GMT	00 and 12 GMT
25,000–30,000	0.59	0.65	0.76	0.53
30,000–34,000	.36	.40	.47	.20
34,000–40,000	.37	.41	.35	.09
Total 25,000–40,000	1.32	1.46	1.58	0.82

free atmosphere that were based on the theory of atmospheric turbulence. Trout and Panofsky (1969) used the clear-air turbulence spectrum measurements and statistics of clear-air turbulence to estimate the energy dissipation near the tropopause. Ellsaesser (1969) used Kolmogorov's structure functions (1941) and Crutcher's upper wind statistics (1959–1962) for a climatological study of the energy dissipation over the Northern Hemisphere. His estimate of the dissipation in the free atmosphere, 3.89 watts m^{-2} over the North American Continent, is very close to our value of 3.50 watts m^{-2} that may be obtained from table 3. Table 8 compares Trout and Panofsky's and Ellsaesser's estimate of the dissipation near the tropopause with that in this study. Dissipation values by Ellsaesser and those of this study were interpolated for the atmospheric layers used by Trout and Panofsky. Considering the difference in approach and data source and uncertainties involved in the data processing of each study, the agreement between their values and 00 GMT values in this study must be recognized to be extremely close. The dissipation values in this study as the mean of the 00 and 12 GMT observations are lower than those at 00 GMT. However, a good overall agreement between their values and our 00 and 12 GMT average still should be recognized. As discussed in section 2, it is difficult to present positive evidence against the diurnal variation, from the technical viewpoints of our computation scheme. Wallace and Hartranft (1969) reported that their observation of the direction of the diurnal tide in three atmospheric layers below 30 km qualitatively agrees with our diurnal variation of $-\overline{\mathbf{V}\cdot\nabla\phi}$ in those layers. Nevertheless, our observation of significant magnitude of the diurnal variation of energy parameters is open to question and comments are invited.

Data coverage for our study has been limited to north of 25° N. According to Ellsaesser's climatological estimate (1969), although the magnitude is somewhat smaller than in middle latitudes, an appreciable amount of the energy dissipation in the free atmosphere was also indicated in the Tropics. A strong recent argument by Charney (1968) concerning the existence of the upper Ekman boundary layer is particularly noteworthy, both in connection with the energy dissipation in the Tropics and the problem of dissipation in the free atmosphere in general.

5. CONCLUDING REMARKS

With the improved scheme for computing the horizontal and vertical transports of kinetic energy, we have shown, on a firmer basis than before, that a considerable amount of the kinetic energy is dissipated outside the planetary boundary layer, that there are two maxima of generation and dissipation in the boundary layer and at the jet-stream level, and that the dissipation value is much higher than currently assumed in most numerical models of the atmosphere.

This report concludes our preliminary study of the kinetic energy balance. Currently, a continued work is in progress with the support of the National Science Foundation (NSF Grant GA–1287) in an effort to extend the energy balance study to other regions of the Northern Hemisphere, to examine the three-dimensional distributions of the energy balance, and also to examine the problem of the energy cascade with a mesoscale radiosonde observation network.

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